AD-A086 163 MASSACHUSETTS INST OF TECH CAMBRIDGE OPERATIONS RESE--ETC F/G 5/1
HIERARCHICAL PRODUCTION PLANNING: A SINGLE STAGE SYSTEM.(U)
MAR 80 G R BITRAN, E A HAAS, A C HAX N00014-75-C-0556
UNCLASSIFIED TR-174 NL

1 OF 1
AD
A086163

END
DATE
FILMED
8-80
DTIC

HIERARCHICAL PRODUCTION PLANNING: A SINGLE STAGE SYSTEM

by

GABRIEL R. BITRAN,

ELIZABETH A. HAAS

and

ARNOLDO C. HAX

Technical Report No. 174

OPERATIONS RESEARCH CENTER

MASSACHUSETTS INSTITUTE
OF
TECHNOLOGY

March 1980

6

HIERARCHICAL PRODUCTION PLANNING: A SINGLE STAGE SYSTEM,

by

10

GABRIEL R./BITRAN

ELIZABETH A./HAAS

[redacted]

ARNOLDO C./HAX

14) TR-174

9) Technical Report No. 174

15) Work Performed Under

Contract N00014-75-C-0556, Office of Naval Research

Multilevel Logistics Organization Models

Project No. NR 347-027

Operations Research Center

Massachusetts Institute of Technology

Cambridge, Massachusetts 02139

11) Mar[redacted]980

12) 45

Dist | Avail and/or special
A

270720

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: Technical Report No. 174 — 2. GOVT ACCESSION NO.: AD-A086 163	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): HIERARCHICAL PRODUCTION PLANNING: A SINGLE STAGE SYSTEM	5. TYPE OF REPORT & PERIOD COVERED: Technical Report March 1980
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Gabriel R. Bitran, Elizabeth A. Haas, Arnoldo C. Hax	8. CONTRACT OR GRANT NUMBER(s): N00014-75-C-0556
9. PERFORMING ORGANIZATION NAME AND ADDRESS: M.I.T. Operations Research Center, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR 347-027
11. CONTROLLING OFFICE NAME AND ADDRESS: O.R. Branch, ONR Navy Department, 800 North Quincy Street, Arlington, VA 22217	12. REPORT DATE: March 1980
	13. NUMBER OF PAGES: 41 pages
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report)
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Releasable without limitation on dissemination.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Aggregate Production Planning
Production Planning
Disaggregation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

See page ii

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported, in part, by government contracts and grants. The work reported herein was supported by the Office of Naval Research under Contract No. N00014-75-C-0556.

Richard C. Larson
Jeremy F. Shapiro

Co-Directors

ABSTRACT

This paper presents a hierarchical approach to plan and schedule production in a manufacturing environment that can be modelled as a single stage process. Initially, the basic trade-offs inherent to production planning decisions are represented by means of an aggregate model, which is solved on a rolling horizon basis. Subsequently, the first solution of the aggregate plan is disaggregated, considering additional cost objectives and detailed demand constraints.

Several improvements in the methodology related to hierarchical production planning are suggested. Special attention is given to alternative disaggregation procedures, problems of infeasibilities, and the treatment of high setup costs. Computational results, based on real life data, are presented and discussed.

1. INTRODUCTION

In this paper we will concentrate exclusively on the design of model-based systems to support tactical and operational decisions pertinent to production planning. Tactical decisions are concerned with the allocation of the resources available for production purposes. Typical decisions are the amounts of each product type to be produced in each period, the levels of regular and overtime workforce to be used, specification of service levels, inventory targets, machines capacities, etc. An appropriate planning horizon for these decisions is a full seasonal cycle to capture the fluctuations in demand due to seasonalities and promotions. The length of the cycle is usually one year. To make effective tactical decisions it is generally sufficient to consider aggregate data. The basic operational decisions of production planning consist in establishing the amount of each item that must be produced in each period and the corresponding resources needed. Operational decisions are made subject to the limitations imposed by the tactical level and require a high degree of detailed information.

In many practical situations of batch processing systems, tactical and operational decisions are taken by distinct managerial echelons. This fact must be recognized by system designers if an implementation is to succeed. Although production planning has attracted the attention of operation researchers for a long time, the different nature of the two classes of decisions mentioned above has been seldomly considered. Some exceptions include the works of Holt, Modigliani, Muth and Simon [16], Winters [27], Hax and Meal [15], Bitran and Hax [2], Shwimer [23], Newson [21], and Zoller [28]. However, most publications encountered in the literature either formulate the production planning problem at a detailed level, [7],[8],[19],[20], or advocate an aggregate approach and give little insight into how to disaggregate the solutions, see [4],[5],[11],[18], and [25]. The detailed formulation of the problem leads to a very large mathematical programming problem which is very difficult to interact with, requires demand data that can seldomly be forecasted with an acceptable accuracy, and is expensive to operate.

In this paper we present an improved version of the hierarchical procedure for production planning suggested by Bitran and Hax [2] and provide theoretical results supporting the method. To make this paper as self sufficient as possible we briefly describe the hierarchical production planning concept.

1.1 Hierarchical Production Planning

Hax and Meal introduced the concept of hierarchical production planning in [15]. The method consists primarily of recognizing the differences between tactical and operational decisions. The tactical decisions are associated with aggregate production planning while the operational decisions are an outcome of the disaggregation process.

Hax and Meal proposed the following levels of aggregation:

Items: are the end products delivered to customers.

Product Types: are groups of items having similar unit costs, direct costs (excluding labor), holding costs per unit per period, productivities (number of units that can be produced per unit of time), and seasonalities.

Families: are groups of items pertaining to a same product type and sharing similar setups. That is, whenever a machine is prepared to produce an item of a family, all other items in the same family can also be produced with a minor change in setups.

Although we have adopted this three-level product structure in our work, the reader should realize that a specific disaggregation hierarchy depends on the actual setting being considered.

An overview of hierarchical production planning is shown in Figure 1.1. It applies to single stage batch production processes. An extension for multistage processes is given in [1]. As indicated in Figure 1.1, three levels, paralleling the aggregation hierarchy, are recommended (boxes 1, 2, and 3). The first is the product type level where aggregate plans for product types are determined. The planning horizon is usually one year. This level is concerned with tactical decisions. Only the amount of each product type to be produced in the first period is passed to the family level (box 2). Aggregate production plans are determined on a rolling horizon basis. That is, the planning horizon is maintained equal to one year by deleting the last period and adding a new one. At the second level the run quantity of each type is disaggregated to obtain the production quantities of each family. These are passed to the item level (box 3) where they are further disaggregated to determine the amount of each item to be produced in the first period. It is important to notice that in the hierarchical method detailed forecasts at the item level over the entire aggregate planning horizon are not needed. As a consequence, the data collection required is considerably smaller than in detailed formulations of the production planning problem. Moreover, the fact that there are usually a few number of product types justifies the use of sophisticated forecasting techniques that would be prohibitively expensive to employ for thousands of items. Since aggregate forecast tends to be more accurate than detailed forecast, production plans generated by hierarchical planning tend to be quite stable in the rolling horizon process.

Hax and Meal proposed a heuristic to perform the three levels (boxes 1, 2, and 3) in Figure 1.1. Bitran and Hax formalized the hierarchical planning

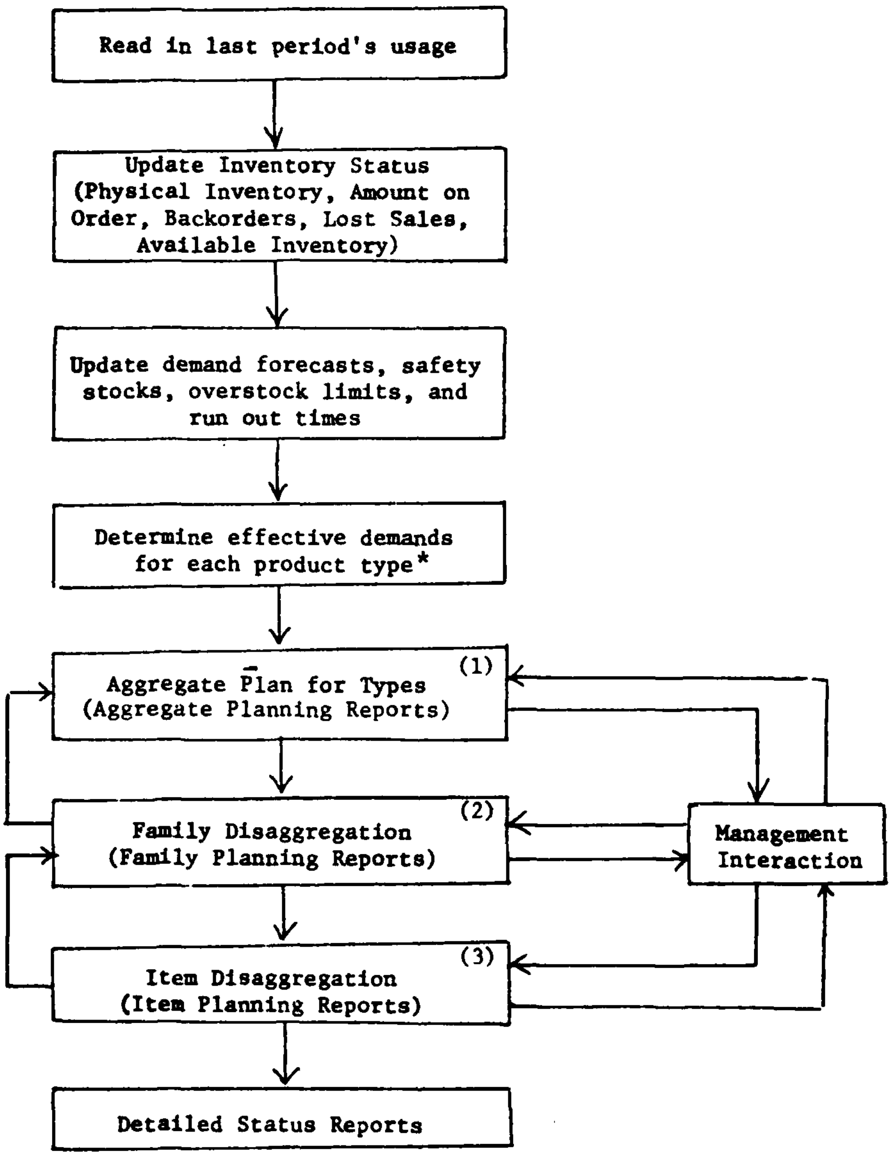


Figure 1.1: Conceptual Overview of Hierarchical Planning System

* For a discussion of effective demands, see Section 2.2.

heuristic by suggesting the use of convex knapsack problems to disaggregate the product type and families run quantities into families and item run quantities, respectively.

The plan of the paper is as follows. In section 2 the algorithm suggested by Bitran and Hax is briefly reviewed. In section 3 theoretical results supporting the hierarchical production planning method are provided. Three modifications to the algorithm in [2] are introduced in section 4. Results of extensive computational experiments comparing several hierarchical procedures are given in section 5. Conclusions are presented in section 6. Proofs of the theorems are presented in the appendices.

2. HIERARCHICAL PRODUCTION PLANNING - THE REGULAR KNAPSACK METHOD

The first formal system that represents our philosophy for hierarchical production planning was proposed by Hax and Meal [15]. A formalization of that approach was developed by Bitran and Hax [2]. In this section we will summarize the basic features of the Bitran and Hax approach, which will be referred to as the Regular Knapsack Method (RKM). The origin of this name is due to the fact that the family and item disaggregation subsystems are both represented by means of Knapsack problems.

2.1 Aggregate Production Planning for Product Types

Aggregate production planning, the highest level of planning in hierarchical production planning systems, addresses product type scheduling. The following linear program provides a simple representation of that planning problem.

Problem (P)

$$\text{minimize} \quad \sum_{i=1}^{I} \sum_{t=1}^{T} (c_{it}X_{it} + h_{it}I_{it}) + \sum_{t=1}^{T} (r_tR_t + o_tO_t)$$

$$\begin{aligned}
\text{subject to} \quad I_{it-1} + X_{it} - I_{it} &= d_{it} && i=1,2,\ldots,I;\ t=1,2,\ldots,T.\\
\sum_{i=1}^{I} m_iX_{it} &\leqq R_t + O_t && t=1,2,\ldots,T.\\
R_t &\leqq (rm)_t && t=1,2,\ldots,T.\\
O_t &\leqq (om)_t && t=1,2,\ldots,T.\\
X_{it}, I_{it}, R_t, O_t &\geqq 0 && i=1,2,\ldots,I;\ t=1,2,\ldots,T.
\end{aligned}$$

The decision variables of the model are: X_{it}, the number of units to be produced of type i during period t; $I_{i,t}$, the number of units of inventory of type i left over at the end of period t; and R_t and O_t, the regular hours and the overtime hours used during period t, respectively.

The parameters of the model are: T, the length of the planning horizon; c_{it}, the unit production cost (excluding labor); h_{it}, the inventory carrying cost per unit per period; r_t and o_t, the cost per manhour of regular labor and of overtime labor; $(rm)_t$ and $(om)_t$, the total availability of regular and overtime hours in period t, respectively; m_i, the hours required to produce one unit of product type i; and $d_{i,t}$, the effective demand for type i during period t. (A definition of effective demand is given below.) For simplicity of presentation, we have not incorporated in Problem (P) hiring and firing, production lead time, backorders and other features that can be easily considered. Moreover, the aggregate Problem (P) need not necessarily be formulated as a linear programming problem. Any aggregate production planning model may be used as long as it adequately represents the practical setting under consideration. Discussions of aggregate models can be found in [12],[6],[22], and [17]. Problem (P) is solved with a rolling horizon of length T. At the end of every time period, new information becomes available and is used to update the values of the parameters of the model, particular demand forecasts, and the resulting values of the decision variables. Due to the uncertainties present in the planning process, only first period results of the aggregate problem are actually implemented.

2.2 Effective Demand

As is shown in Bitran and Hax [2], the use of demand, instead of effective demand, may lead to an aggregate solution of Problem (P) which cannot be disaggregated. The effective demand represents net requirements which cannot be satisfied from initial available inventory. More precisely, let I_{ko} represent the initial inventory of item k, and $\bar{d}_{kt}$ denote the demand of item k in period t. The effective demands of item k are defined as:

$$d_{kt} = \begin{cases} \max(0, \sum_{\tau=1}^{t} \bar{d}_{k\tau} - I_{ko}^{i}) & \text{if } d_{kt-1}=0 \text{ (define } d_{ko}=0\text{) and} \\ \bar{d}_{kt} & \text{otherwise.} \end{cases}$$
(t=1,2,..,T)

These computations force one to provide detailed forecasts for each item until the initial inventory is exhausted. The effective demand of product type i is defined as the sum of the effective demands of all of its items, i.e.:

$$d_{it} = \sum_{k\varepsilon K(i)} d_{kt}$$

where K(i) is the set of all items belonging to product type i.

Notice that since, in Problem (P), d_{it} represents effective demand it follows that the initial inventory of every product type i, I_{io} is equal to zero.

2.3 A Family Disaggregation Model

Since only the first period's results of the aggregate production problems are to be implemented, the family disaggregation model attempts to allocate the production quantities X_{i1} of each product type to the families belonging to that type. The disaggregation is performed using, for each product type i, the following continuous knapsack problem which determines run quantities for each family attempting to minimize the total setup cost among families.

Problem (Pi)

$$\text{Minimize} \quad \sum_{j\varepsilon J(i)} \frac{s_j d_j}{Y_{j1}}$$

$$\text{subject to} \quad \sum_{j\varepsilon J(i)} Y_{j1} = X_{i1}$$

$$\ell b_{j1} \leq Y_{j1} \leq ub_{j1} \qquad j\varepsilon J(i)$$

where s_j and d_j denote the setup cost and annual demand of family j. The variables Y_{j1} represent the quantity to be produced of each family j during period 1. The upper bound ub_{j1} and lower bound ℓb_{j1} are computed as follows:

$$ub_{j1} = \max(0, os_{j1} - ai_{j1}) \text{ and}$$

$$\ell b_{j1} = \max(0, \bar{d}_{j1} - ai_{j1} + ss_{j1})$$

where os_{j1}, d_{j1}, ai_{j1}, and ss_{j1} denote respectively the overstock limit, the demand, the available inventory, and the safety stock of family j in period 1. J(i) is the set of families in product type i that trigger in period 1, i.e., it is the set of indices j such that $\bar{d}_{j1} - ai_{j1} + ss_{j1} > 0$.

The objective function of Problem (Pi) assumes that the family run quantities are proportional to the setup cost and the annual demand for a given family. This assumption, which is the basis of the economic order quantity formulation, tends to minimize the average annual setup cost. In section 4.1 this objective function will be reviewed. An efficient algorithm to solve Problem (Pi) is given in Bitran and Hax [2].

2.4 The Item Disaggregation Model

Once the quantities Y_{j1} have been determined, one needs to disaggregate them among the items belonging to each family j. For the current planning period, all costs have already been determined by the two previous stages in the hierarchical process. However, the feasible solution chosen will establish the intial conditions of the next period and will affect future costs. In order to save setups in future periods, it seems reasonable to distribute the family run quantity among its items in such a way that each item's runout time coincides with the runout time of the family. This can be accomplished by the following continuous knapsack problem.

Problem (Qj)

$$\text{Minimize} \quad \sum_{k\epsilon K(j)} \left[\frac{Y_{j1} + \sum_{k\epsilon K(j)} (ai_{k1}-ss_{k1})}{\sum_{k\epsilon K(j)} d_{k1}} - \frac{Z_k + ai_{k1} - ss_{k1}}{d_{k1}} \right]^2$$

$$\text{subject to} \quad \sum_{k\epsilon K(j)} Z_{k1} = Y_{j1}$$

$$\ell b_{k1} \leq Z_{k1} \leq ub_{k1} \qquad k\epsilon K(j)$$

The variables Z_{k1} denote the production quantity of item k in family j. K(j) is the set of items in family j and the parametes d_{k1}, ai_{k1}, ss_{k1}, ℓl_{k1}, and ub_{k1} represent for item k the same quantities that were discussed for family j in Problem (Pi). An efficient algorithm to solve Problem (Qj) is presented in [2].

3. COMPARING DISAGGREGATION PROCEDURES

An important determinant for the performance of hierarchical production planning systems is the procedure used to disaggregate earlier decisions at each hierarchical level. The knapsack approach just presented identifies one possible alternative for hierarchical designs. The knapsack nature of the subproblems is very appealing because of its great computational advantage. However, it is imperative that we gain some theoretical understanding of the impact of different disaggregation schemes on the production planning costs. Such an understanding will help us in evaluating and comparing alternative disaggregation mechanisms, and in judging the improvements to be obtained by introducing modifications to the RKM.

This section will describe two fundamental theorems which provide important insights into the strengths of various disaggregation methodologies. Let the superscript u denote a generic disaggregation procedure applied to Problem (P). The effective demands can be expressed as a function of the real demands (or forecasted demands) and the disaggregation procedure used as follows:

$$d_{it} = \bar{d}_{it} - g^u_{it} \qquad \text{for } i=1,2,\ldots,I \text{ and } t=1,2,\ldots,T.$$

The quantity g^u_{it} represents the total contribution of all items belonging to product type i to determine the effective demand of that product type in period t, using the disaggregation procedure u. Different disaggregation procedures will affect the initial inventory of each item and, thus, the effective demand of the product types. Therefore, $\sum_{t=1}^{\tau} g^u_{it}$, $\tau=1,\ldots,T$ indicate the sum of the real or forecasted demands of the items in product type i that can be satisfied directly by the initial inventory up to period τ.

Problem (P) can be rewritten as a function of disaggregation u as follows:

Problem (P^u)

$$z^u = \min \sum_{i=1}^{I} \sum_{t=1}^{T} (c_{it}X_{it}^u + h_{it}I_{it}^u) + \sum_{t=1}^{T} (r_t R_t^u + o_t O_t^u) +$$

$$+ \sum_{i=1}^{I} \sum_{t=1}^{T-1} h_{it} \sum_{k=t+1}^{T} g_{ik}^u$$

$$\text{subject to} \quad I_{it-1}^u + X_{it}^u - I_{it}^u = d_{it} - g_{it}^u \qquad i=1,2,\ldots,I;\ t=1,2,\ldots,T.$$

$$\sum_{i=1}^{I} m_i X_{it}^u \leq R_t^u + O_t^u \qquad t=1,2,\ldots,T.$$

$$R_t^u \leq (rm)_t \qquad t=1,2,\ldots,T.$$

$$O_t^u \leq (om)_t \qquad t=1,2,\ldots,T.$$

$$X_{it}^u, I_{it}^u, R_t^u, O_t^u \geq 0 \qquad i=1,2,\ldots,I;\ t=1,2,\ldots,T.$$

where the last term in the objective function represents the holding cost of the initial inventory of all product types, which is a function of the disaggregation procedure used. This term has been omitted in Problem (P) since, given a disaggregation procedure, the initial inventory cost is constant. However, it is important to include it in Problem (P^u) because our purpose is to compare the performance of various disaggregation procedures.

The quantities g_{it}^u satisfy the following condition

$$\sum_{t=1}^{T} g_{it}^u = I_{io} \qquad i=1,2,\ldots,I$$

where I_{io} is the initial inventory of product type i.

A disaggregation procedure u is said to be feasible if Problem (P^u) is feasible.

<u>Theorem 3.1</u>: Let u_1 and u_2 be two generic feasible disaggregation procedures such that

$$\sum_{k=1}^{t} g_{ik}^{u_1} \geqq \sum_{k=1}^{t} g_{ik}^{u_2} \qquad i=1,2,\ldots,I;\ t=1,2\ldots,T. \tag{3.1}$$

Then $z^{u_1} \leqq z^{u_2}$.

The proof of this theorem is provided in Appendix 1, and its implications are discussed below.

Define u=0 to be the case where $g_{i1}^{o} = I_{io}$ and $g_{it}^{o} = 0$, $i=1,2,\ldots,I$; $t=2,\ldots,T$. This implies the allocation of all initial inventory to the first period of the planning horizon, even if the demands during that period become negative. Moreover, since $g_{i1}^{o} = I_{io} = \sum_{k=1}^{T} g_{ik}^{u_j}$ for $i=1,2,\ldots,I$ and for every feasible disaggregation u_j, it follows that

$$\sum_{i=1}^{t} g_{ik}^{o} \geqq \sum_{k=1}^{t} g_{ik}^{u_j}$$

for every $t=1,2,\ldots,t$; $i=1,2,\ldots,I$ and every u_j. Hence, if we let z^o be the optimal value of problem (P^o) corresponding to u=0 the corollary below is directly implied by theorem 3.1.

<u>Corollary 3.1</u>: Assume (P^o) is feasible. Then, $z^o \leqq z^{u_j}$ for every feasible disaggregation u_j.

Theorem 3.1 provides a mechanism for comparing two disaggregation procedures u_1 and u_2. If conditions 3.1 are met, u_1 is preferred to u_2. In this sense, conditions 3.1 can be viewed as establishing a partial order structure in the space of feasible disaggregations.

The key question to ask at this point is whether a feasible disaggregation procedure exists which is optimal, i.e., that attains the lower bound z^o. The answer to the question is provided by Theorem 3.2 below which establishes the conditions under which the disaggregation procedure known as "Equalization of Run Out Times" (EROT) becomes optimal.

EROT allocates the production amount determined at the aggregate planning level for a given product type, X_{i1}, in such a way as to equalize the run out times of all the items belonging to that type. The run out time for a product type is defined as the number of periods (possibly a fractional number) that will elapse until the inventory of that item reaches the safety stock level.

Theorem 3.2: Let u_E denote the EROT disaggregation procedure. Then, if all disaggregations are made with the EROT disaggregation method, if $I_{io} \geq 0$ for all product types, and if the aggregate product type problem (P^o) is feasible it follows that

$$z^o = z^{u_E}$$

The proof to this theorem is provided in Appendix 2.

Recognizing that the aggregate plan ignores setup costs, there are two important conclusions that are derived from Theorems 3.1 and 3.2.

First, a qualitative interpretation of Theorem 3.1 is that the larger the g_{it}'s are for small values of t, the better it is in terms of total primary costs. That is, the best disaggregation scheme must allocate the initial inventory of each product type uniformly, in terms of runout time, among all its items. This interpretation makes clear that the EROT disaggregation is optimal, as proved in Theorem 3.2. However, when setup costs grow in importance it is desirable to allocate larger inventories to the items that have the highest setup costs. The motivation for a non-uniform allocation of inventories among items in a product type is the desire to minimize total setup costs. Therefore, we face a tradeoff between using the EROT disaggregation procedure and consequently minimizing total primary costs while incurring high setup costs, and choosing a disaggregation that allocates the inventories in a manner that considers setup cost levels reducing these costs while incurring higher primary costs. Our computational experience, to be discussed in section 5, has shown that when the total setup costs are 5% or less of the total production costs, EROT performs quite efficiently. The RKM, enhanced by modifications to be introduced in the next section, is one of those methodologies that are more effective than EROT when setup costs are significant.

Second, EROT is a "myopic" disaggregation rule. The aggregate planning model covers a long planning horizon, usually a full year, to allow for an efficient allocation of facilities, manpower, and inventory under fluctuating demand conditions. However, once the aggregate plan has been established, we need just to look for a few periods ahead (the length of which is represented by the runout time) to determine an appropriate disaggregation. Although EROT is an optimal disaggregation procedure only under the absence of setup costs, the qualitative implications of this approach support the essence of hierarchical planning, that advocates the use of long planning horizons at the highest planning level, while drastically decreasing the planning horizons at the lower planning levels.

4. MODIFICATIONS INTRODUCED IN THE REGULAR KNAPSACK METHOD

The concerns about the performance of disaggregation procedures under high setup costs, and the myopic nature of disaggregation rules, led us to incorporate modifications in the Regular Knapsack Method. Computational results evaluating the performance of EROT, RKM, and the modified Knapsack will be presented in section 5. This section will cover three important changes made to RKM.

4.1 A Myopic Objective Formulation for the Family Subproblem

The objective function proposed originally in the RKM (see section 2.3) was:

$$\text{Minimize} \sum_{j \in J(i)} \frac{s_j d_j}{Y_{j1}} .$$

The definition of d_j, covering demand over the entire planning horizon, contradicts the myopic nature of the disaggregation process referred to in the previous section. As a consequence, the resulting solutions Y_{j1} of Problem (Pi) will not be sensitive to the difference among the demand patterns of the families belonging to a given product type in the immediate future. Inspired by the results of Theorem 3.1 and 3.2, d_j was redefined as a demand for family j over the run out time of its corresponding product type; that is to say, over the time interval that would exhaust the production quantity X_{i1} of the product type i containing family j when that quantity is disaggregated according to the EROT scheme.

4.2 The Look Ahead Feasibility Rule

Another important problem that led us into modifying the RKM was the generation of infeasibilities that could be introduced in the aggregate problem by the disaggregation procedure used. Golovin [10] detected this problem and illustrated its occurance by means of a numerical example. Gabbay [9] suggested a set of constraints to be introduced in the family subproblem that would provide necessary and sufficient conditions for the existence of feasible disaggregations over the entire planning horizons. His results, however, were restricted to the static case; that is, when Problem (P) is solved only at the beginning of the first period and no rolling horizon is used. Moreover, Gabbay's constraints destroy the special knapsack structure of the family subproblems, thus eliminating the computational advantages of such a structure.

To overcome the potential presence of infeasibilities, we developed a simple rule that looks ahead just one period, attempting to prevent the next period's disaggregation from becoming infeasible. We designated this rule as the "Look Ahead Feasibility Rule". The essence of the computations needed to carry out this rule is presented in Figure 4.1.

A numerical example might facilitate an understanding of the applications of this rule.

Assume that the aggregate schedule for product type P1, composed by families F1 and F2, is:

Product type P1:	Initial Inventory	Production	Demand	Ending Inventory
Period 1	10 units	25 units	20 units	15 units
Period 2	15 units	5 units	20 units	0 units

Demand and inventory data for families F1 and F2 are as follows:

	F1	F2
Initial inventory in period 1	0 units	10 units
Demand in period 1	10 units	10 units
Demand in period 2	10 units	10 units

The inventory figures do not include safety stocks.

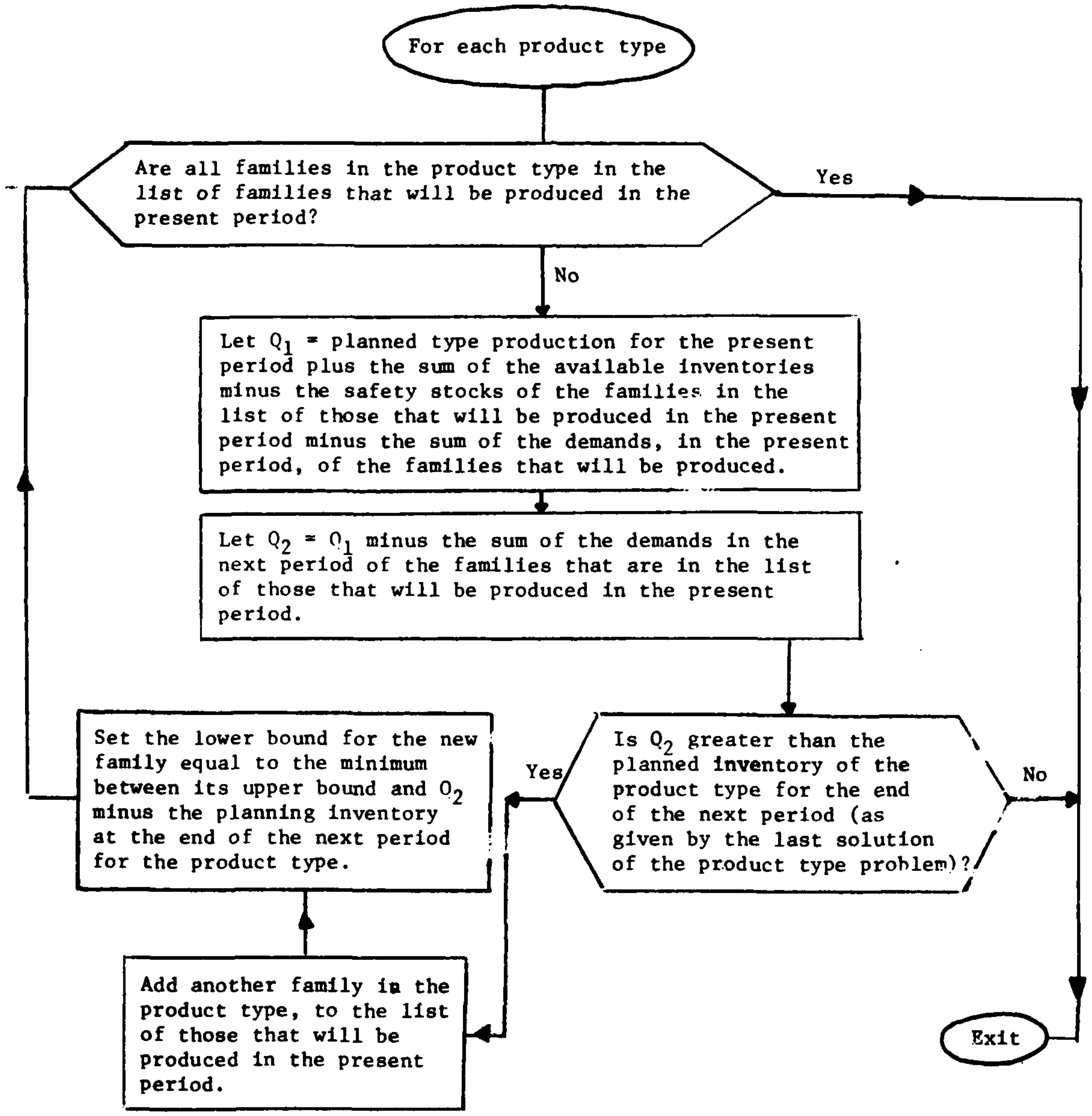


Figure 4.1: The Look Ahead Feasibility Rule

According to this data, only F1 will trigger in the first period. Therefore, under RKM, only F1 will be produced and its production quantity will be 25 units (for simplicity, we are assuming no upper bounds for both families). In the second period, only F2 will trigger, since its initial inventory will be zero units and its demand will be 10 units. However, since the production scheduled for P1 is 5 units, a shortage of 5 units for family 2 will results. If the Look Ahead routine in Figure 4.1 is applied, the value of Q_1 will be $Q_1 = 25-0-10 = 15$ and $Q_2 = 15-10 = 5$. The planned inventory for the product type P1 at the end of period 2 will be zero units. Hence, $Q_2 = 5 > 0$. Therefore, the "look ahead feasibility rule" adds to the list of families to be produced in the present period (period 1), composed just by F1, the family F2 with a lower bound of 5 units for its production quantity. This modification will eliminate the infeasibility created by RKM. It is important to note that this adjustment routine does not preclude the use of the efficient knapsack algorithm to solve the family subproblem (Pi).

4.3 Modification of the Regular Knapsack Method for the Case of High Setup Costs

We have already addressed the role of setup costs in hierarchical production planning systems. The initial approaches introduced by Hax and Meal [15] and Bitran and Hax [2] ignore the setup costs at the product type level, and include them in the decision rules at the family level. The resulting algorithms proved to be effective when setup costs did not exceed 10 percent of the total production costs (for further discussion of this subject see [2],[14]).

The issues that still deserve consideration are those cases in which setup costs represented a percentage higher than 15 of the total production cost. Figure 4.2 describes a subroutine that we introduced to the RKM for those situations with fairly high setup costs. This routine can be easily modified to allow for managerial inputs which reflect their judgment regard-

For each product type i solve the family subproblem (Pi)

For each family j, compute the integer number of periods, N(j), of demand that can be satisfied with the production quantity y_{j1} allocated to the family by subproblem (Pi).

For each family j, compute the "ideal production quantity", using Silver-Meal lot sizing procedure [24], and the associated integer number of periods, M(j), of demand that can be satisfied by this production quantity.

For each family j, free the production capacity corresponding to producing more than min [N(j); M(j)] periods of demand.

Let CAP denote the sum of the production capacities freed by each family independently of the product type it belongs to.

Allocate the capacity CAP to the family(ies) in quantities equal to full periods of demand, whenever possible, in such a way to "minimize the total production costs".

If the aggregate problem (P) did not use all regular hours of production in the present period, treat this extra capacity as free capacity and allocate it to some family(ies) as long as it has a positive impact on the total costs.

Figure 4.2: Routine to Adapt the RKM for the Case of High Setup Costs

ing changes to be incorporated in the aggregate schedule in order to save setup costs. These changes will invariably represent tradeoffs between the linear costs identified in the aggregate production level and the setup costs incurred at the family level.

The routine briefly described in Figure 4.2 works as follows. Initially the family subproblems (Pi) are solved and the production quantities Y_{j1} for each family are obtained. The integer number of periods of demand, for each family j, that can be satisfied by Y_{j1} is computed and denoted by N(j). Next, Silver-Meal's lot sizing method [24] is applied, for each family j, considering the stream of demands starting with the present period. The otuput of the method is denoted by L(j) and we refer to it as the ideal quantity to be produced for family j in the present period. We have chosen Silver-Meal's procedure instead of the Wagner-Whitin [26] algorithm because the first is much more efficient computationally and gives satisfactory results (see [22] for a comparison of the two methods). The next step is to compute the integer number M(j) of periods of demand that can be totally satisfied by L(j). It is important to note that N(j), L(j), and M(j) are computed considering effective family demands. For each family, the difference between the capacity allocated by the family subproblems and the capacity needed to cover the demand for more than minimum [M(j),N(j)] periods is considered freed. The sum of the freed capacities of each family, independently of the product type to which it belongs, is denoted by Z. After removing the free capacity from each family, we denote the remaining production quantity by Q(j). All families are ordered according to "increasing marginal costs", $MC(j) = -\frac{1}{Q(j)}\left[\frac{s_j d_j}{Q(j)} - \frac{h_i Q(j)}{2}\right]$, where s_j is the setup cost for family j, d_j is the demand of family j over the myopic planning horizon (as defined in section 4.1), and h_i is the cost of holding one unit in stock for one period. The capacity Z is then allocated to the families in quantities equal to full periods of demand (whenever possible, starting with the one with minimum MC(j)). After allocating one period of demand to a family j, its marginal cost is recomputed with Q(j) increased by the corresponding amount. If after allocating the capacity Z there exists at least one family with negative marginal cost, and there are regular hours of production available, that the aggregate problem (P) did not use for the present period, the routine allocates the time available to those families. We would like to point out that variants of the marginal criterion, MC(j), have been tested and none performed better than the one reported here. This approach effectively alters the aggregate schedule as long as the expected savings in setup costs more than compensate for changes in the costs considered by the aggregate schedule.

5. COMPUTATIONAL RESULTS

A series of experiments were conducted to examine the performance of the modifications introduced in the hierarchical knapsack method and to compare this method with others. The data used for these tests was taken from a manufacturer of rubber tires. The product structure characteristics, together with relevant information, are given in Figure 5.1. The twelve items were partitioned into two product types P1 and P2. Product type P1 is composed by two families P1F1 and P2F2. The second product type is partitioned into three families P2F1, P2F2, and P2F3. Table 5.1 exhibits the demand pattern for both product types.

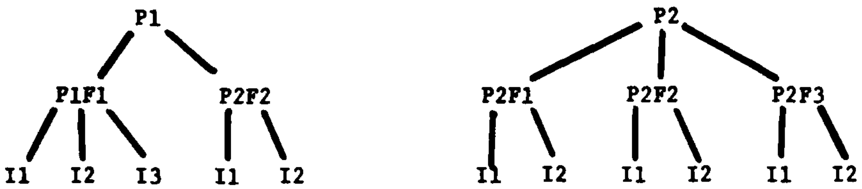
The experiments were divided in two sets. In the first set, they consisted of applying the production planning methods to a full year of simulated plant operations. Production decisions were made every four weeks. The model was then updated using a one year planning horizon. The process was repeated thirteen times. At the end of the simulation, total setup costs, inventory holding costs, overtime costs, and backorders were calculated. Direct manufacturing costs and regular work force costs were omitted because they were considered fixed costs for this applications.

The methods compared are:

1) The RKM modified by considering the myopic horizon for the demand at the family level (subsection 4.1) and the "look ahead feasibility rule" (subsection 4.2). In Table 5.2 this method corresponds to the column K12.

2) The RKM with the three modifications, i.e., the modifications considered in 1) above, plus the adjustment routine for high setup costs (subsection 4.3). In Table 5.2 this method corresponds to the column K123.

3) EROT which consists in disaggregating the product type production quanti-



Family setup cost = 90
Holding cost = $.31/unit a month
Overtime cost = $9.5/hour
Productivity factor = .1 hr/unit
Production lead time = 1 month

Family setup cost = $120
Holding cost = $.40/unit a month
Overtime cost = $9.5/hour
Productivity factor = .2 hrs/unit
Production lead time = 1 month

Regular Workforce Costs and Unit Production Costs are considered fixed costs.

Total Regular Worforce = 2000 hrs/month
Total Overtime Workforce = 1200 hrs/month

Figure 5.1: Product Structure and Relevant Information

Table 5.1: Demand Patterns of Product Types

Time Period t	Product Type 1 P1	Product Type 2 P2
1	12,736	6,174
2	7,813	2,855
3	0	4,023
4	0	4,860
5	0	7,131
6	0	9,665
7	1,545	17,603
8	7,895	14,276
9	10,982	11,706
10	15,782	15,056
11	16,870	8,232
12	15,870	7,880
13	9,878	10,762
TOTAL	99,371	120,223

ties directly into item quantities using as a criterion the equalization of the run out times, i.e., the production quantity of each product type is allocated among the items in such a way that they last for an equal number of periods (assuming perfect forecast). In Table 5.2 this method corresponds to the column EROT.

Seventy two experiments were performed. The base case corresponds to the data taken from the manufacturer of rubber tires. The data for the other seventy one experiments was constructed by perturbing the data of the base case in order to explore the effects of the production capacity, magnitude, and relative values of the setup costs and forecast errors. Global statistics will be provided for the seventy two experiments. Due to space limitations, we report in Table 5.2 the results of seventeen problems solved. This subset is representative of the results obtained throughout the seventy two experiments performed in so far as identifying meaningful combinations of available capacities, forecast errors, and setup costs. For the purpose of comparison, we have also solved the seventeen problems by the RKM. The corresponding results are shown in column RKM in Table 5.2. The data structure used in the computational experiments is given below.

Capacity (3 cases)

C1 : 2000 hrs/month regular time

C2 : 2500 hrs/month regular time

C3 : 1600 hrs/month regular time

Overtime is 60% of the regular hours in all three cases.

<u>Forecast Errors</u> (3 cases)

F1 : zero forecast error

F2 : $.02 + .01t^{1.3}$ with no bias

F3 : $.02 + .01t^{1.3}$ all positive forecast errors

where t denotes the period in the planning horizon of the aggregate problem. F2 and F3 assume that the forecast error increases in absolute value as t increases. That is, the further away a period is, the higher the average absolute value of the forecast error is. For F2, the probabilities of positive and negative forecast errors were assumed to be equal to .5.

<u>Setup Costs</u> (8 cases)

		Product type 1	Product type 2
S1	Family 1	90	110
	Family 2	90	110
	Family 3	-	110
S2	Family 1	900	1100
	Family 2	900	1100
	Family 3	-	1100
S3	Family 1	900	110
	Family 2	1800	110
	Family 3	-	110
S4	Family 1	500	1000
	Family 2	2000	100
	Family 3	-	50
S5	Family 1	5000	400
	Family 2	50	400
	Family 3	-	1000
S6	Family 1	3000	110
	Family 2	4000	110
	Family 3	-	110
S7	Family 1	6000	400
	Family 2	4500	5000
	Family 3	-	3000
S8	Family 1	300	300
	Family 2	90	100
	Family 3	-	400

The notation used in Table 5.2 is as follows: CiFjSk indicates that the capacity data used is Ci, the forecast error structure if Fj, and the setup cost structure is Sk. C1F1S1 corresponds to the base case.

Some conclusions that can be drawn from the computational experiments are:

1) Independent of capacity limitations and forecast errors, under low setup costs, K123 does not perform as effectively as K12. However, as expected, under high setup costs the routine is effective and should be used.

2) Although the backorders observed are not significant in the experiments performed, they are always lower in K12 and K123 than in the EROT method. In all cases with tight capacity and forecast error (either biased or unbiased) the EROT procedure carried backorders.

3) All three methods react as expected to high forecast errors and capacity constraints.

4) The cases in which the EROT procedure performed better than the other two methods were characterized by extremely low setup costs. However, the improvement over K12 is not significant even in those few cases.

5) In thirteen out of the seventeen cases, K12 outperformed RKM in terms of total cost. In the four cases where the reverse occurs, the regular hierarchical knapsack method presents a significant number of backorders.

6) It is interesting to observe that except in one case with significant forecast error (C3F2S7), the sum of holding and overtime costs are smaller for the EROT than for other methods. This fact is a direct consequence of Theorem 3.2.

To test if the observed differences in the total costs obtained with the four methods are statistically significant, we performed Wilcoxon's

Table 5.2: A Comparison of the Costs Resulting When the Various Approaches Were Tested on Sample Points

CASE	COST TYPE*	RKM	K12	K123	EROT	BEST M.I.P. SOLUTION FOUND
Base Case C1F1S1	Holding	29920	30651	45476	29923	
	Setup	5580	5360	5030	5910	
	Overtime	81684	81113	72319	81681	
	Total	117184	117120	122825	117514	115616
	Setup/Total	4.8%	4.6%	4.1%	5.0%	
	% Difference in Cost from M.I.P.	1.4	1.3	6.2	1.6	
	Backorders	-	-	-	-	
C1F1S8	Holding	31221	33195	30739	29923	
	Setup	11910	12610	12210	13910	
	Overtime	82382	79302	81682	81682	
	Total	125513	125107	124631	125515	122790
	Setup/Total	9.5%	10.1%	9.8%	11.1%	
	% Difference in Cost from M.I.P.	2.2	1.9	1.5	2.2	
	Backorders	71 units	-	-	-	
C1F1S5	Holding	31922	32028	35921	29923	
	Setup	67050	67050	54650	68850	
	Overtime	81682	80926	79714	81681	
	Total	180654	180004	170285	180454	165550
	Setup/Total	37.1%	37.2%	32.1%	38.2%	
	% Difference in Cost from M.I.P.	9.1	8.7	2.9	9.0	
	Backorders	4 units	-	-	-	
C2F1S5	Holding	13583	14042	47784	13584	
	Setup	67850	67050	53850	68850	
	Overtime	48577	48878	24681	47578	
	Total	130010	129970	126315	130012	124236
	Setup/Total	52.2%	51.6%	42.6%	53.0%	
	% Difference in Cost from M.I.P.	4.9	4.6	1.7	4.6	
	Backorders	-	-	-	-	

CASE	COST TYPE*	RKM	K12	K123	EROT
C1F2S1	Holding	64380	65473	65386	63806
	Setup	4740	5070	5030	5730
	Overtime	80957	78907	78657	79709
	Total	150077	149450	152073	149245
	Setup/Total	3.2	3.4	3.3	3.8
	Backorders	5 units	-	-	-
C1F3S1	Holding	83034	86315	94553	78197
	Setup	4300	5180	5070	5910
	Overtime	88396	83491	80303	90412
	Total	175730	174986	179926	174519
	Setup/Total	2.5	3.0	2.8	3.4
	Backorders	-	-	-	-
C1F3S7	Holding	85343	90295	89088	78197
	Setup	194000	171300	152200	203700
	Overtime	88753	77729	85709	90412
	Total	368096	339324	326997	372309
	Setup/Total	52.7	50.5	46.5	54.7
	Backorders	6 units	-	-	-
C2F1S1	Holding	13584	13584	21849	13584
	Setup	5910	5910	5310	5910
	Overtime	48878	47578	42237	47578
	Total	68371	67072	68396	67072
	Setup/Total	8.6	8.8	7.8	8.8
	Backorders	-	-	-	-
C2F1S7	Holding	13583	14066	59739	13584
	Setup	200500	195300	134100	203700
	Overtime	48878	48878	28047	47578
	Total	262961	258244	221881	264862
	Setup/Total	76.2	75.6	60.4	76.9
	Backorders	-	-	-	-

CASE	COST TYPE*	RKM	K12	K123	EROT
C2F2S1	Holding	49856	49856	53721	49856
	Setup	5730	5730	5400	5910
	Overtime	50269	48523	47503	48523
	Total	105855	104109	106624	104289
	Setup/Total	5.4	5.5	5.1	5.7
	Backorders	-	-	-	-
C3F1S1	Holding	73016	76157	82455	77584
	Setup	3930	4480	4480	5910
	Overtime	118560	117326	111320	112560
	Total	195506	197963	198255	196054
	Setup/Total	2.0	2.3	2.3	3.0
	Backorders	5522 units	-	-	-
C3F1S7	Holding	71584	92752	92678	77584
	Setup	198500	192700	172100	203700
	Overtime	118560	98960	99830	112560
	Total	388645	384412	364608	393844
	Setup/Total	51.1	50.1	47.2	51.7
	Backorders	6942 units	-	-	-
C3F2S1	Holding	84620	100304	106213	80986
	Setup	4300	4920	4810	5910
	Overtime	118560	92930	88600	108243
	Total	207480	198154	199623	195139
	Setup/Total	2.1	2.5	2.4	3.0
	Backorders	8159 units	301 units	301 units	1412 units
C3F2S7	Holding	84842	97526	108409	82986
	Setup	202900	195300	178700	203700
	Overtime	118560	91634	91450	108243
	Total	406302	384460	378559	392929
	Setup/Total	49.9	50.8	47.2	51.8
	Backorders	6178 units	-	-	-

CASE	COST TYPE*	RKM	K12	K123	EROT
C3F3S1	Holding	87030	120364	117426	86926
	Setup	4300	4920	4920	5910
	Overtime	118560	87192	90257	118531
	Total	209900	212476	212603	211367
	Setup/Total	2.1	2.3	2.3	2.8
	Backorders	10302 units	-	-	-
C3F3S7	Holding	86570	120349	145205	86926
	Setup	190600	188800	174700	203700
	Overtime	118559	92219	74385	118531
	Total	395729	400368	394290	409157
	Setup/Total	48.2	47.2	44.3	49.8
	Backorders	10158 units	-	-	-
C3F1S5	Holding	71584	90084	90826	77584
	Setup	60050	50450	54650	68850
	Overtime	118559	104864	100560	112560
	Total	250193	255398	246038	258994
	Setup/Total	24.0	19.8	22.2	26.6
	Backorders	6942 units	-	-	-

* CODE:

Holding Cost	- dollars
Setup Cost	- dollars
Overtime Cost	- dollars
Total Cost	- dollars
Setup/Total	- percentage
Backorders	- units

signed rank test. The test was used to pairwise compare the methods. The null hypothesis is that the total costs of the first approach are less than or equal to those of the second. Table 5.3 shows the results obtained for the Wilcoxon's test. WI is the Wilcoxon statistics and σ is its standard deviation.

Table 5.3: Results Obtained for the Wilcoxon's Test

Methods Compared	Wilcoxon Statistics	Sample Size	Confidence with which null hypothesis can be rejected
RKM vs. K12	WI = 1.81σ	17	96%
RKM vs. K123	WI = 1.90σ	17	97%
K12 vs. K123	WI = 5.36σ	72	>99%
EROT vs. K123	WI = 5.53σ	72	>99%

The Wilcoxon statistics indicate that overall, the adjusted knapsack with feedback is superior to all other approaches. However, our detailed analysis suggests that if the setup costs are very small, i.e., less than 10% of the total cost, the feedback algorithm should not be used.

The second set of experiments consisted in solving a selected sample of the seventy two problems as mixed integer programming problems (MIP). The formulation of the production planning problems as MIP's can be seen as an optimal representation. The four problems shown in Table 5.4 were solved using the Land and Powell package on the computer Prime 400 at M.I.T. Unfortunately, although each problem contains only sixty five zero-one variables, no "true optimal" solution was found within forty hours of connect time for each of the four problems. In Table 5.4 we indicate the best solution available at time of interruption of the computer programs. Due to the poor performance of the mixed integer package we limited the experiments to only four problems which were solved just once (rather than on a rolling horizon basis). This last fact favors the MIP formulations. To facilitate the comparison between the methods, the results corresponding to the four problems obtained in Table 5.2 for the RKM, K12, K123, and EROT algorithms are repeated in Table 5.4.

Table 5.4: Total Costs

	C1F1S1	C1F1S5	C1F1S8	C2F1S5
RKM	117184	180654	125513	130010
K12	117120	180004	125107	129970
K123	122825	170285	124631	126315
EROT	117514	180454	125515	130012
Best MIP Solution	115616	165550	122790	124236

The results in Table 5.4 indicate that when the setup costs are less than 5% of total costs and K12 is used, or when the setup costs are greater than 5% and K123 is used, the total annual costs were never more than 3% greater than the best MIP solution found after forty hours of connect time. Finally we point out that none of the seventy two problems solved by the RKM, K12, K123, and EROT algorithms on a rolling horizon basis, i.e. solved thirteen times over the horizon of one year, exceeded ten minutes of connect time on the M.I.T. computer Prime 400.

6. CONCLUSIONS

The experimentation reported herein tends to confirm our belief that hierarchical planning systems provide a very effective alternative for supporting production planning decisions at a tactical and operational level. When contrasted with a mixed integer programming formulation, hierarchical planning methods produce near optimal solutions with significantly smaller computational efforts and data collection requirements. The hierarchical planning approach represents a feasible alternative for the solution of large scale real life problems which will be unthinkable to tackle with an M.I.P. based model. Moreover, and most important from a pragmatic point of view, the hierarchical approach parallels the hierarchy of production planning decisions within the firm.

From a methodological point of view, our experiments seem to indicate that the modifications introduced to the Regular Knapsack Method clearly improve the performance of previous algorithms. The K123 method, under the wide variety of situations tested, outperforms statistically all other methodological alternatives considered. However, a closer examination of those cases where setup costs account for less than ten percent of the total production cost indicates that K12 or EROT might be preferred over K123. EROT and K12 tend to perform quite closely under low setup cost conditions.

ACKNOWLEDGEMENTS

The authors would like to express their gratitude to Josep Valor, for his computational assistance, and Prof. Stephen C. Graves for many insightful comments.

APPENDIX 1: PROOF OF THEOREM 3.1

Theorem 3.1: Let u_1 and u_2 be two generic feasible disaggregation procedures such that

$$\sum_{k=1}^{t} g_{ik}^{u_1} \geqq \sum_{k=1}^{t} g_{ik}^{u_2} \qquad i=1,2,\ldots,I;\ t=1,2,\ldots,T. \tag{3.1}$$

Then $z^{u_1} \leqq z^{u_2}$.

Proof: Let $(X^{u_2}, I^{u_2}, R^{u_2}, O^{u_2})$ be a feasible solution of (P^{u_2}). Hence,

$X_{it}^{u_1} = X_{it}^{u_2}$, $R_t^{u_1} = R_t^{u_2}$, $O_t^{u_1} = O_t^{u_2}$ and $I_{it}^{u_1} = I_{it}^{u_2} + \sum_{k=1}^{t} g_{ik}^{u_1} - \sum_{k=1}^{t} g_{ik}^{u_2}$

$i=1,2,\ldots,I;\ t=1,2,\ldots,T$ is feasible in (P^{u_1}). Moreover the objective function value of (P^{u_1}) for this feasible solution is:

$$z^{u_1} \leqq \sum_{i=1}^{I}\sum_{t=1}^{T}(c_{it}X_{it}^{u_1} + h_{it}I_{it}^{u_1}) + \sum_{t=1}^{T}(r_tR_t^{u_1} + o_tO_t^{u_1}) + \sum_{i=1}^{I}\sum_{t=1}^{T-1} h_{it} \sum_{k=t+1}^{T} g_{ik}^{u_1} =$$

$$= \sum_{i=1}^{I}\sum_{t=1}^{T}(c_{it}X_{it}^{u_2} + h_{it}I_{it}^{u_2}) + \sum_{t=1}^{T}(r_tR_t^{u_2} + o_tO_t^{u_2}) +$$

$$\sum_{i=1}^{I}\sum_{t=1}^{T} h_{it}\left(\sum_{k=1}^{t} g_{ik}^{u_1} - \sum_{k=1}^{t} g_{ik}^{u_2}\right) + \sum_{i=1}^{I}\sum_{t=1}^{T-1} h_{it}\sum_{k=t+1}^{T} g_{ik}^{u_1} =$$

$$= \sum_{i=1}^{I}\sum_{t=1}^{T}(c_{it}X_{it}^{u_2} + h_{it}I_{it}^{u_2}) + \sum_{t=1}^{T}(r_tR_t^{u_2} + o_tO_t^{u_2}) + \sum_{i=1}^{I}\sum_{t=1}^{T-1} h_{it}\sum_{k=t+1}^{T} g_{ik}^{u_2} -$$

$$\sum_{i=1}^{I}\sum_{t=1}^{T-1} h_{it}\sum_{k=t+1}^{T} g_{ik}^{u_2} - \sum_{i=1}^{I}\sum_{t=1}^{T} h_{it}\sum_{k=1}^{t} g_{ik}^{u_2} + \sum_{i=1}^{I}\sum_{t=1}^{T} h_{it}\sum_{k=1}^{t} g_{ik}^{u_1} +$$

$$\sum_{i=1}^{I}\sum_{t=1}^{T-1} h_{it}\sum_{k=t+1}^{T} g_{ik}^{u_1} =$$

$$= \sum_{i=1}^{I}\sum_{t=1}^{T}(c_{it}X_{it}^{u_2} + h_{it}I_{it}^{u_2}) + \sum_{t=1}^{T}(r_tR_t^{u_2} + o_tO_t^{u_2}) + \sum_{i=1}^{I}\sum_{t=1}^{T-1} h_{it}\sum_{k=t+1}^{T} g_{ik}^{u_2}$$

(A1.1)

The last inequality in (3.2) follows from the fact that $\sum_{k=1}^{T} g_{ik}^{u_1} = \sum_{k=1}^{T} g_{ik}^{u_2} = I_{io}$ i=1,2,...,I. The conclusion that can be drawn from (A1.1) is that the optimal value of problem (P^{u_1}) is not higher than the value of the objective function of problem (P^{u_2}) for any feasible solution in (P^{u_2}) that is, $z^{u_1} \leqq z^{u_2}$.

Theorem 3.1 indicates that any two feasible disaggregation procedures are not necessarily comparable in terms of total aggregate production costs. It is important to note however that the optimal value z^u of (P^u) used to compare disaggregation schemes is a proxy for the total production cost of the hierarchical method.

APPENDIX 2: PROOF OF THEOREM 3.2

Theorem 3.2: Let u_E denote the EROT disaggregation procedure. Then, if all disaggregations are made with the EROT disaggregation method, if $I_{io} \geqq 0$ for all product types, and if the aggregate product type problem (P^o) is feasible it follows that

$$z^o = z^{u_E}$$

Proof: Every summation $\sum_{t=a}^{b}$ with $b < a$ is defined as being zero. Let u_E denote the EROT disaggregation procedure and assume that all disaggregations are made using this method. Let $I_{io} \geqq 0$, $i=1,2,\ldots,I$, be the initial inventory of product type i before the computations of the effective demands. Recall that safety stocks are not included in the I_{io} for $i=1,2,\ldots,I$. Denote by $\bar{d}_{it}$ the real demand of product type i in time period t for $i=1,2,\ldots,I$ and $t=1,2,\ldots,T$. Since the EROT disaggregation procedure is being used each inventory I_{io} will last for R(i) periods where

$$R(i) = r(i) + \frac{I_{io} - \sum_{t=1}^{r(i)} \bar{d}_{it}}{\bar{d}_{ir(i)+1}} \qquad i=1,2,\ldots,I \tag{A2.1}$$

r(i) is the smallest nonnegative integer satisfying

$$I_{io} - \sum_{t=1}^{r(i)+1} \bar{d}_{it} < 0 \ .$$

Moreover, the product type problem (P^{u_E}) that we need to solve at the beginning of period 1 is such that the $g_{it}^{u_E}$ are nonnegative and satisfy for each $i=1,2,\ldots,T$ the condition $I_{io} = \sum_{t=1}^{T} g_{it}^{u_E}$. The first term in (A2.1) is the smallest integer less than or equal to R(i).

The EROT disaggregation method implies that for each product type $i=1,2,\ldots,I$ one of two following cases occur:

a) If $r(i) \geqq 1$ then $g_{it}^{{}^{u}E} = \bar{d}_{it}$, $t=1,2,\ldots,r(i)$, and (A2.2)

$$0 \leqq I_{io} - \sum_{k=1}^{r(i)} \bar{d}_{ik} = g_{ir(i)+1}^{{}^{u}E} \leqq \bar{d}_{ir(i)+1} \tag{A2.3}$$

b) If $r(i) < 1$ then $I_{io} = g_{i1}^{{}^{u}E} < \bar{d}_{i1}$ (A2.4)

Assume that (P^o) is feasible and that $(X^o,I^o,R^o,0^o)$ is one of its optimal solutions. Define

$$X^{{}^{u}E} = X^o,\ R^{{}^{u}E} = R^o,\ 0^{{}^{u}E} = 0^o \text{ and } I_{it}^{{}^{u}E} = \sum_{k=1}^{t} X_{ik}^o - \sum_{k=1}^{t} \bar{d}_{ik} + \sum_{k=1}^{t} g_{ik}^{{}^{u}E}$$

$$t=1,2,\ldots,T;\ i=1,2,\ldots,I. \tag{A2.5}$$

To show that $(X^{{}^{u}E},I^{{}^{u}E},R^{{}^{u}E},0^{{}^{u}E})$ is feasible in $(P^{{}^{u}E})$ we still need to prove that it satisfies the mass balance constraints and that $I^{{}^{u}E} \geqq 0$.
First we prove that the mass balance constraints hold at $(X^{{}^{u}E},I^{{}^{u}E},R^{{}^{u}E},0^{{}^{u}E})$.

$$I_{it-1}^{{}^{u}E} + X_{it}^{{}^{u}E} - I_{it}^{{}^{u}E} = \sum_{k=1}^{t-1} X_{ik}^o - \sum_{k=1}^{t-1} \bar{d}_{ik} + \sum_{k=1}^{t-1} g_{ik}^{{}^{u}E} + X_{it}^o - \sum_{k=1}^{t} X_{ik}^o + \sum_{k=1}^{t} \bar{d}_{ik} - \sum_{k=1}^{t} g_{ik}^{{}^{u}E}$$

$$= \bar{d}_{it} - g_{it}^{2} \qquad i=1,2,\ldots,I;\ t=1,2,\ldots,T. \tag{A2.6}$$

the first equality in (A2.6) follows from (A2.5).
Next, we prove that $I^{{}^{u}E} \geqq 0$. Note that

1) for $1 \leqq t \leqq r(i)$, $I_{it}^{{}^{u}E} = \sum_{k=1}^{t} X_{ik}^o \geqq 0$, $i=1,2,\ldots,I$ by (A2.2)

2) for $t = r(i)+1$

$$I_{ir(i)+1}^{{}^{u}E} = \sum_{k=1}^{r(i)+1} X_{ik}^o - \sum_{k=1}^{r(i)+1} \bar{d}_{ik} + \sum_{k=1}^{r(i)+1} g_{ik}^{{}^{u}E} =$$

$$= \sum_{k=1}^{r(i)+1} X_{ik}^o - \sum_{k=1}^{r(i)+1} \bar{d}_{ik} + I_{io} = I_{ir(i)+1}^o \geqq 0 \qquad i=1,2,\ldots,I;$$

the second equality follows from (A2.2), (A2.3), and (A2.4) since they imply that $I_{io} = \sum_{k=1}^{r(i)+1} g_{ik}^{u_E}$;

3) for $t > r(i)+1$,

$$I_{it}^{u_E} = \sum_{k=1}^{t} X_{ik}^{o} - \sum_{k=1}^{t} \bar{d}_{ik} + \sum_{k=1}^{t} g_{ik}^{u_E} = \sum_{k=1}^{t} X_{ik}^{o} - \sum_{k=1}^{t} \bar{d}_{ik} + I_{io} =$$

$$= I_{it}^{o} \geqq 0. \quad i=1,2,..,I;$$

the second equality follows from (A2.2), (A2.3), and (A2.4) since they imply that $I_{io} = \sum_{k=1}^{r(i)+1} g_{ik}^{u_E}$ and hence $g_{ik}^{u_E} = 0$, $k=r(i)+2,\ldots,T$.

From 1), 2), and 3) we have that $(X^{u_E}, I^{u_E}, R^{u_E}, O^{u_E})$ is feasible in (P^{u_E}).

Therefore,

$$z^{u_E} \leqq \sum_{i=1}^{I} \sum_{t=1}^{T} (c_{it} X_{it}^{u_E} + h_{it} I_{it}^{u_E}) + \sum_{t=1}^{T} (r_t R_t^{u_E} + o_t O_t^{u_E}) + \sum_{i=1}^{I} \sum_{t=1}^{T-1} h_{it} \sum_{k=t+1}^{T} g_{ik}^{u_E} =$$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} [c_{it} X_{it}^{o} + h_{it} (\sum_{k=1}^{t} X_{ik}^{o} - \sum_{k=1}^{t} \bar{d}_{ik} + \sum_{k=1}^{t} g_{ik}^{u_E})] +$$

$$+ \sum_{t=1}^{T} (r_t R_t^{u_E} + o_t O_t^{u_E}) + \sum_{i=1}^{I} \sum_{t=1}^{T-1} h_{it} \sum_{k=t+1}^{T} g_{ik}^{u_E} =$$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} [c_{it} X_{it}^{o} + h_{it} (\sum_{k=1}^{t} X_{ik}^{o} - \sum_{k=1}^{t} \bar{d}_{ik} + I_{io})] + \sum_{t=1}^{T} (r_t R_t^{o} + o_t O_t^{o}) +$$

$$+ \sum_{i=1}^{I} \sum_{t=1}^{T-1} h_{it} \sum_{k=t+1}^{T} g_{ik}^{u_E} - \sum_{i=1}^{I} \sum_{t=1}^{T} h_{it} (I_{io} - \sum_{k=1}^{t} g_{ik}^{u_E}) =$$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} (c_{it} X_{it}^{o} + h_{it} I_{it}^{o}) + \sum_{t=1}^{T} (r_t R_t^{o} + o_t O_t^{o}) = z^{o}$$

hence, $z^{u_E} \leqq z^{o}$.

However, from corollary 3.1, $z^{o} \leqq z^{u_E}$. Consequently $z^{o} = z^{u_E}$.

References

1. Bitran, G. R., E. Haas, and A. C. Hax, "Hierarchical Production Planning: A Multi Stage System", M.I.T., Operations Research Center, Technical Report No. , 1980.

2. Bitran, G. R., and A. C. Hax, "On the Design of Hierarchical Production Planning Systems", Decision Sciences, Vol. 8, No. 1, January 1977, pp. 28-55.

3. Bitran, G. R., and A. C. Hax, "On the Solution of Convex Knapsack Problems with Bounded Variables", M.I.T., Operations Research Center, Technical Report No. 129, April 1977.

4. Bowman, E. H., "Production Scheduling by the Transportation Method of Linear Programming", Operations Research, Vol. 4, No. 1, February 1956, pp. 100-103.

5. Bowman, E. H., "Consistency and Optimality in Managerial Decision Making", Management Science, Vol. 9, No. 2, January 1963, pp. 310-321.

6. Buffa, E. S., and J. G. Miller, Production-Inventory Systems: Planning and Control, Irwin, 1979.

7. Dzielinski, B. P., C. T. Baker, and A. S. Manne, "Simulation Tests of Lot Size Programming", Management Science, Vol. 9, No. 2, January 1963, pp. 229-258.

8. Dzielinski, B. P., and R. E. Gomory, "Optimal Programming of Lot Sizes, Inventory and Labor Allocations", Management Science, Vol. 11, No. 9, July 1965, 874-890.

9. Gabbay, H., "A Hierarchical Approach to Production Planning", M.I.T., Operations Research Center, Technical Report No. 120, December 1975.

10. Golovin, J. J., "Hierarchical Integration of Planning and Control", M.I.T., Operations Research Center, Technical Report No. 116, September 1975.

11. Hanssmann, F., and S. W. Hess, "A Linear Programming Approach to Production and Employment Scheduling", Management Technology, No. 1, January 1960, pp. 46-51.

12. Hax, A. C., "Aggregate Production Planning", in Handbook of Operations Research, J. Moder and S. E. Elmaghraby (eds.), Van Nostrand Reinhold, 1978.

13. Hax, A. C., "The Design of Large Scale Logistics Systems: A Survey and an Approach", in Modern Trends in Logistics Research, W. H. Marlow (ed.), MIT Press, 1976.

14. Hax, A. C., and J. J. Golovin, "Hierarchical Production Planning Systems", in Studies in Operations Management, A. C. Hax (ed.), North Holland, 1978.

15. Hax, A. C., and H. C. Meal, "Hierarchical Integration of Production Planning and Scheduling", in Studies in Management Sciences, Vol. I, Logistics, M. A. Geisler (ed.), North Holland-American Elsevier, 1975.

16. Holt, C. C., F. Modigliani, J. F. Muth, and H. A. Simon, Planning Production, Inventories and Work Force, Prentice Hall, 1960.

17. Johnson, L. A., and D. C. Montgomery, Operations Research in Production Planning, Scheduling and Inventory Control, John Wiley, 1974.

18. Jones, C. H. "Parametric Production Planning", Management Science, Vol. 13, No. 11, July 1967, pp. 843-866.

19. Lasdon, L. S., and R. C. Terjung, "An Efficient Algorithm for Multi-Item Scheduling", Operations Research, Vol. 19, No. 4, 1971, pp. 946-969.

20. Manne, A. S., "Programming to Economic Lot Sizes", Management Science, Vol. 4, No. 2, 1958, pp. 115-135.

21. Newson, E. P., "Multi-Item Lot Size Scheduling by Heuristic, Part I: With Fixed Resources, Part II: With Variable Resources", Management Science, Vol. 21, No. 10, July 1975.

22. Peterson, R., and E. A. Silver, Decision Systems for Inventory Management and Production Planning, John Wiley, 1979.

23. Shwimer, J., "Interactions Between Aggregate and Detailed Scheduling in a Job Shop", unpublished Ph.D. Thesis, Alfred P. Sloan School of Management, M.I.T., June 1972.

24. Silver, E. A., and H. C. Meal, "A Heuristic for Selecting Lot Size Quantities for the Case of a Deterministic Time Varying Demand Rate and Discrete Opportunities for Replenishment", Production and Inventory Management, second quarter, 1973.

25. Taubert, W. H., "A Search Decision Rule for the Aggregate Scheduling Pattern", Management Science, Vol. 14, No. 6, February 1968.

26. Wagner, H. M., and T. M. Whitin, "Dynamic Version of the Economic Lot Size Model", Management Science, Vol. 5, No. 1

27. Winters, P. R., "Constrained Inventory Rules for Production Smoothing", Management Science, Vol. 8, No. 4, July 1962.

28. Zoller, K., "Optimal Disaggregation of Aggregate Production Plans", Management Science, Vol. 17, No. 8, April 1971, p;. B533-B547.